OPERATIONS RESEARCH FOR PATIENT – CENTERED HEALTH CARE DELIVERY

Proceedings of the XXXVI International ORAHS Conference

edited by
Angela Testi, Elena Tànfani,
Enrico Ivaldi, Giuliana Carello,
Roberto Aringhieri,
Vito Fragnelli

FrancoAngeli

ECONOMIA - *Ricerche*

OPERATIONS RESEARCH FOR PATIENT – CENTERED HEALTH CARE DELIVERY

Proceedings of the XXXVI International ORAHS Conference

18-23 July, 2010
Genova

edited by
Angela Testi, Elena Tànfani,
Enrico Ivaldi, Giuliana Carello,
Roberto Aringhieri,
Vito Fragnelli

FrancoAngeli

INDEX

PREFACE

Governance and regulation of health services are major issues in contemporary societies because demand is raising both in terms of quantity and quality. At the same time, governments face increasing difficulties in finding adequate financial resources to meet health care needs. The debate on the Health Care Systems is focusing on resource use, with the aim of reducing resource wasting, and, as a consequence, of reducing overall service cost. However, the quality of the services provided to citizens cannot and must not be neglected: health care management must pay attention to citizens and patients needs and expectations. Such scenario proposes tremendous challenges to health care policy makers, administrators and practitioners. Careful analysis, management, optimization and evaluation are needed to simultaneously meet economic and quality requirements.

With its more than thirty-year activity, ORAHS, the EURO working group on Operational Research Applied to Health Services, provides a wide knowledge and expertise in quantitative methods applied to health care planning and management. ORAHS members come mainly from Europe, but also from other non European countries. During the years, ORAHS members have cooperated with many hospitals and health care institutions, providing not only theoretical studies, but also real life applied solutions, with a significant attention to the real features of the considered problems.

Every year ORAHS members meet together during the annual ORAHS conference, which is open also to non members, to discuss problems and compare approaches. The conference takes place every year in a different country: the description and analysis of the local Health Care System and of local excellence experiences may provide further discussion topics and challenges.

The XXXVI ORAHS conference took place in Genova in July 2010. The theme of the conference suggests a complete re-organization of all steps in healthcare delivery, to become a “Patient-Centered Healthcare Delivery” to reply to some dissatisfaction coming from the discrepancy between the people’s needs and the system’s answers. The book collects a selection of 33 among the more than 100 contributions presented at the Conference, providing a significant overview on the quantitative studies on health care applications.

In particular, the book covers the main topics highlighted during the conference, such as epidemiology, disease modelling, health policy, planning health services, emergency medical services, home care, nursing homes, logistics, operations scheduling, and quality.

The book is divided into 8 sections. The first section is devoted to epidemiology studies and disease modelling. The second section is devoted to health policy issues. The third section collects contributions dealing with the planning of health care systems. Fourth and fifth sections focus on particular areas of the health systems, emergency medical services and home care delivery and nursing homes, respectively. Sixth and seventh sections are devoted to operational issues: the first is devoted to logistic in health care while the second deals with operations scheduling. Finally, last section is devoted to quality and appropriateness topics.

EDITED BY

Angela Testi, Associate Professor of Political Economy and Health Economics at the Department of Economics and Quantitative Methods (DIEM), University of Genova, Italy.

Elena Tanfani, Assistant Professor of Operations Research at the Department of Economics and Quantitative Methods (DIEM), University of Genova, Italy.

Enrico Ivaldi, PhD researcher at the Department of Economics and Quantitative Methods (DIEM), University of Genova, Italy.

Giuliana Carello, Assistant Professor of Operations Research at the Department of Electronics and Information (DEI), Politecnico of Milano, Italy.

Roberto Aringhieri, Assistant Professor of Computer Science and Operations Research at the Department of Computer Science, University of Torino, Italy.

Vito Fragnelli, Associate Professor of Game Theory at the Department of Advanced Sciences and Technologies (DiSTA), University of Piemonte Orientale, Italy.

EPIDEMIOLOGY AND DISEASE MODELLING

OPTIMAL CONTROL APPLIED TO A DISCRETE INFLUENZA MODEL

Paula González-Parra[1], *Leticia Velázquez*[2], *María C. Villalobos*[3], *Carlos Castillo-Chávez*[4]

Abstract: *A discrete time SIR model is proposed for analyzing the influenza dynamics for a given population over a time interval. The total population is divided into susceptible (S), infected (I), and recovered (R) individuals. Assuming limited resources, the model is expanded in order to evaluate the potential effect of control measures such as treatment and social distancing. By using 1918 influenza pandemic data and optimal control with interior-point methods, our goal is to estimate the fraction of treatment and social distancing in order to minimize the number of infected individuals.*

Keywords: *Disease policy modelling, nonlinear programming, patients.*

1. Introduction

Every year millions of individuals get influenza viruses and thousands die as a consequence of them (Hyman et al., 2003). In spring of 2009, an influenza pandemic of type A/H1N1 arose and spread around the world provoking much concern and fear. Type A/H1N1 symptoms, such as fever, lethargy, lack of appetite, and coughing, are similar to seasonal influenza, while additional symptoms may include sore throat, nausea, vomiting, and diarrhea. The 1918 flu pandemic, known as the "Spanish flu", was also caused by the type A/H1N1 virus. This pandemic caused between 20 and 100 million deaths in the world and affected all ages, sexes, and races. Different control measures, such as treatment and social distancing, were applied but their effects have not been studied.

[1] Ph.D. Student, Program in Computational Science, The University of Texas at El Paso. Professor Universidad Autónoma de Occidente, Cali, Colombia.
E-mail: pagonzalezparra@miners.utep.edu.
[2] Associate Professor, Department of Mathematical Sciences, The University of Texas at El Paso.
[3] Associate Professor, Department of Mathematics, The University of Texas Pan American, Edinburg, USA.
[4] Professor, Mathematical and Computational Modelling Sciences Center, Arizona State University, Tempe, USA.

In terms of disease modelling, different continuous time approaches have been used to study the spread of influenza (Nuno et al., 2008). In our case we formulate a discrete time model to study the influenza pandemic. A discrete model is used because data sets are collected in discrete time and it is easy to compare data with the output of a discrete model (Brauer et al., 2009). The main focus of this paper is to study the impact of control policies, such as social distancing and treatment, in an influenza model using the pandemic data recorded from 1918-1919 in San Francisco, California.

This paper is organized as follows. In Section 2 we present the discrete time model (Gonzalez-Parra et al., 2009) in order to study the influenza dynamics. In Section 3 we present the nonlinear programming problem formulation of the influenza model. Moreover, we present a technique used to solve this problem as an optimal control with the addition of interior-point methods. Finally we present some encouraging numerical results.

2. Discrete SIR model

We consider a discrete time model where the total population is divided into susceptible (*S*), infectious (*I*), and recovered (*R*) individuals. The model is given by the system of difference equations:

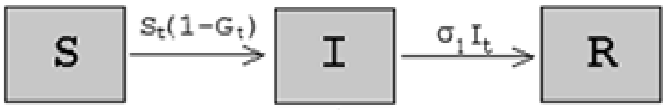
$$\begin{aligned} S_{t+1} &= S_t G_t \\ I_{t+1} &= S_t\left(1-G_t\right)+\left(1-\sigma_1\right)I_t \\ R_{t+1} &= R_t+\sigma_1 I_t\,, \end{aligned} \tag{1}$$

where S_t denotes the number of susceptible individuals at time *t* with similar notation for I_t and R_t, and σ_1 is the recovery rate. We now define a term that is used consecutive through the paper.

Definition 2.1. The parameter G_t denotes the fraction of susceptible individuals at time *t* that remains susceptible at time $t+1$.

In this model, we assume that some individuals' recovery occurs at the beginning of the time stage and some susceptible individuals get infected. We ignore the demographic change in the populations and the disease induced deaths. Then expand upon this model to include treated (*T*) individuals, whose details will be given in Section 3.2. In Fig. 2.1 a stage representation of the disease dynamics is presented.

Fig. 2.1 – Scheme of the discrete SIR model



In the absence of interventions or control measures, the basic reproductive number R_0 is defined as the average of secondary cases that a single infected individual causes in a susceptible population. For model (1) this number was given by (Gonzalez-Parra et al., 2009):

$$R_0 = \frac{\beta}{1-(1-\sigma_1)} = \frac{\beta}{\sigma_1} \tag{2}$$

where β represents the contact rate. In particular if R_0 is greater than 1, the disease will spread in a population. Therefore the possibility of an epidemic increases for large values of R_0. In the case of the 1918 influenza pandemic in San Francisco, California, an estimate of the reproductive number was found to lie in the range of 2-3 (Chowell et al., 2009).

Let σ_1 = 1/7 (Tuite et al., 2010), then the calculated value of β is *0.29* from (2). In Fig. 2.2, we simulate the response of the susceptible, recovered, and infected populations versus time from t = 0 to t = 130 days. The x-axis represents the time in days, and the y-axis labels the fraction of individuals for each class (f_S, f_I, f_R) of the total population $N = S + I + R$. Model (1) was solving using Matlab with

$$G_t = e^{-\beta \frac{I_t}{N}}$$

as defined in (Brauer et al., 2009).

Fig. 2.2 – Fraction of susceptible, infected, and recovered individuals

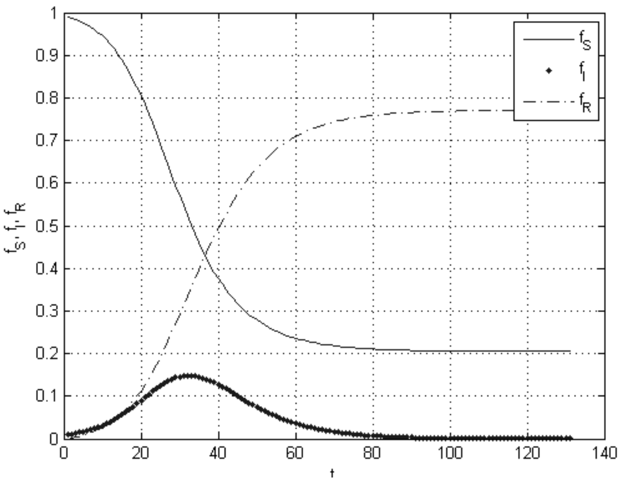
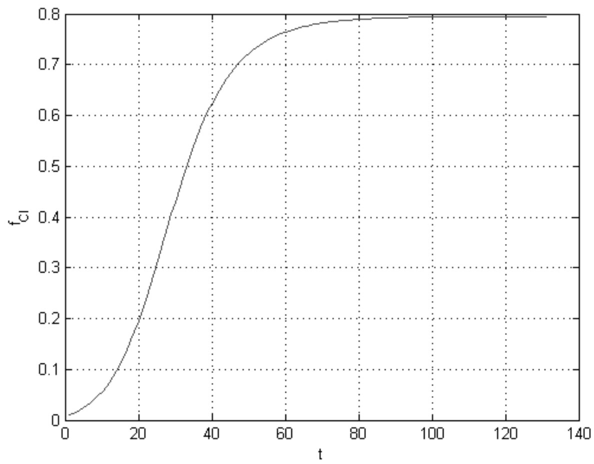


Fig. 2.2 shows that at each time stage the population is constant, i.e. the sum of fractions of susceptible, infected, and recovered individuals is 1. This plot of model (1) demonstrates its effectiveness of simulating the correct behaviour for a single outbreak of influenza.

The next graph, Fig. 2.3, shows the cumulative number of infected individuals. The *y*-axis represents the cumulative fraction of infected individuals (f_{CI}) calculated by (1-f_S). Notice that at the end of the epidemic, 80% of the total population gets the disease, i.e. the final epidemic size is 0.8. We expect this value to be reduced when treatment or social distancing policies are included.

Fig. 2.3 – Cumulative fraction of infected individuals



3. Nonlinear Programming Problem

In this section, we present the optimal control problem of an influenza disease and model it as a nonlinear programming problem.

In order to evaluate the impact of control policies, we include social distancing and treatment in model (1) as the control variables. Our goal is to minimize an objective function $J{:}R^{n+m} \to R$ subject to the difference equations given by the model $y^i_{t+1} \in R^m$ and simple bound constraints for the controls $u^i_t \in R^n$ Then the nonlinear programming problem can be stated as:

$$\begin{aligned} \text{minimize} \quad & J(y^i_t, u^i_t) = \Phi^i(y_{T_f}) + \sum_{t=0}^{T_f - 1} f^i(y^i_t, u^i_t, t) \\ \text{subject to} \quad & y^i_{t+1} = g^i(y^i_t, u^i_t) \\ & 0 \le u^i_t \le u_{\max}, \end{aligned} \tag{3}$$

where the index $i \in \{1,2,3\}$ denotes the selection of the control policy being implemented: (1) social distancing, (2) treatment, and (3) both social distancing and treatment, Tf denotes the final time stage, and t = 0, 1, 2, ..., Tf -1. Table 3.1 provides the definitions for the control variables, state equations and the terms G^i_t (Definition 2.1) for each strategy. The control variables are defined by x_t and τ_t where we let $x_t \in [0, x_{max}]$ denote the amount of social distancing attributed at time t, and $\tau_t \in [0, \tau_{max}]$ denotes the amount of treatment provided at time t.

Tab. 3.1 – Definitions of the control variables, state equations, and G_t^i *for each strategy*

Strategy i	**Control Variables**	**State Equation**	G_t^i
1	$u_t^1 = x_t$	$y_t^1 = (S_t , I_t)$	$G_t^1 = e^{-\beta(1-x_t)\frac{I_t}{N}}$
2	$u_t^2 = \tau_t$	$y_t^2 = (S_t , I_t, T_t)$	$G_t^2 = e^{-\beta\frac{I_t+\rho T_t}{N}}$
3	$u_t^3 = (x_t, \tau_t)$	$y_t^3 = (S_t , I_t, T_t)$	$G_t^3 = e^{-\beta(1-x_t)\frac{I_t+\rho T_t}{N}}$

The functions J and g are defined depending on the control policy being implemented. Next we study the three separate strategies and describe the effects in model (3) as we study each of the controls involved.

3.1. Social Distancing

In this case, we study the impact of implementing social distancing as the control policy in formulation (3). Therefore, the control variable u_t equals $x_{t,}$ which is the reduction in the number of contacts per day. We implement the restriction that we are not allowed to reduce the number of contacts per individual to zero. Our goal is to minimize the objective function $J(y_t^1, u_t^1)$ given in model (3) where the payoff term at the final time T_f is $\Phi^1 = B_4 I_{T_f}$ and the second term whose summands is $f^1 = \frac{1}{2}\left[B_0 I_t^2 + B_2 x_t^2\right]$ for $t = 0$ to $t = T_f - 1$ includes the number of infected individuals and the effort of social distancing at each of the given times. includes the number of infected individuals and the effort of social distancing. The coefficients B_0, B_2, and B_4 are predesigned weights that incorporate the costs of each of the terms in the objective function. The equality constraints y_{t+1}^1 are defined as:

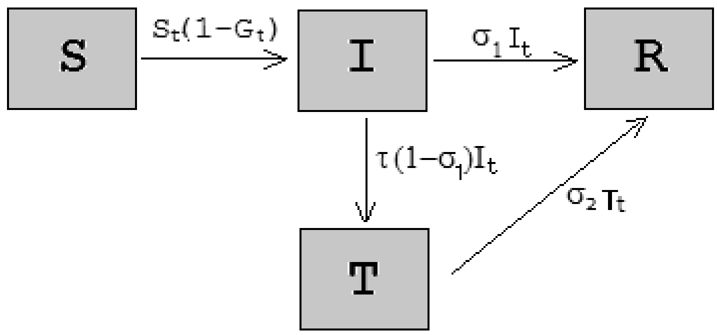
$$g^1(y_t^1, x_t) = \begin{pmatrix} S_t G_t^1 \\ S_t\left(1-G_t^1\right)+\left(1-\sigma_1\right) I_t \end{pmatrix},$$

where G_t^i is given in Table 3.1 and with bound constraints as $0 \leq x_t \leq x_{max}$.

3.2. Treatment

In the case of adding a treatment policy (i = 2), a treated (T) individual class is included in model (1). Let the control variable u_t^2 be τ_t, which is the fraction of the population that receives treatment, ρ is the effective treatment rate, and σ_2 is the recovery rate of treated individuals. Fig. 3.1 shows a stage representation of the disease dynamic with the class of treated individuals.

Fig. 3.1 – Scheme of the discrete model using a treatment policy



In this case, model (1) is replaced by:

$$
\begin{aligned}
S_{t+1} &= S_t G_t^2 \\
I_{t+1} &= S_t\left(1-G_t^2\right)+\left(1-\sigma_1\right)\left(1-\tau_t\right)I_t \\
T_{t+1} &= \left(1-\sigma_2\right)T_t+\tau_t\left(1-\sigma_1\right)I_t \\
R_{t+1} &= R_t+\sigma_1 I_t+\sigma_2 T_t ,
\end{aligned}
\qquad (4)
$$

with G_t^2, specified in Table 3.1.

Our objective is to minimize the objective function $J(y_t^2, u_t^2)$ given in formulation (3) where $\Phi^2 = B_4 I_{T_f} + B_5 T_{T_f}$ is the payoff of the infected and treated individuals at time T_f and the summand $f^2 = \frac{1}{2}\left[B_0 I_t^2 + B_1 T_t^2 + B_3 \tau_t^2\right]$ is taken from $t = 0$ to $t = T_f - 1$. The weights B_i, $i = 0,1,3,4$, and 5 are provided by the user. The equality constraints are given by y_{t+1}^2 with

$$
g^2(y_t^2, \tau_t) = \begin{pmatrix} S_t G_t^2 \\ S_t\left(1-G_t^2\right)+\left(1-\sigma_1\right)\left(1-\tau_t\right)I_t \\ \left(1-\sigma_2\right)T_t+\tau_t\left(1-\sigma_1\right)I_t \end{pmatrix}, \qquad (5)
$$

and the inequality constraints are $0 \le \tau_t \le \tau_{\max}$. Notice that we do not include the last equation corresponding to recovered individuals in model (4) because it does not play a role in the transmission of the disease.