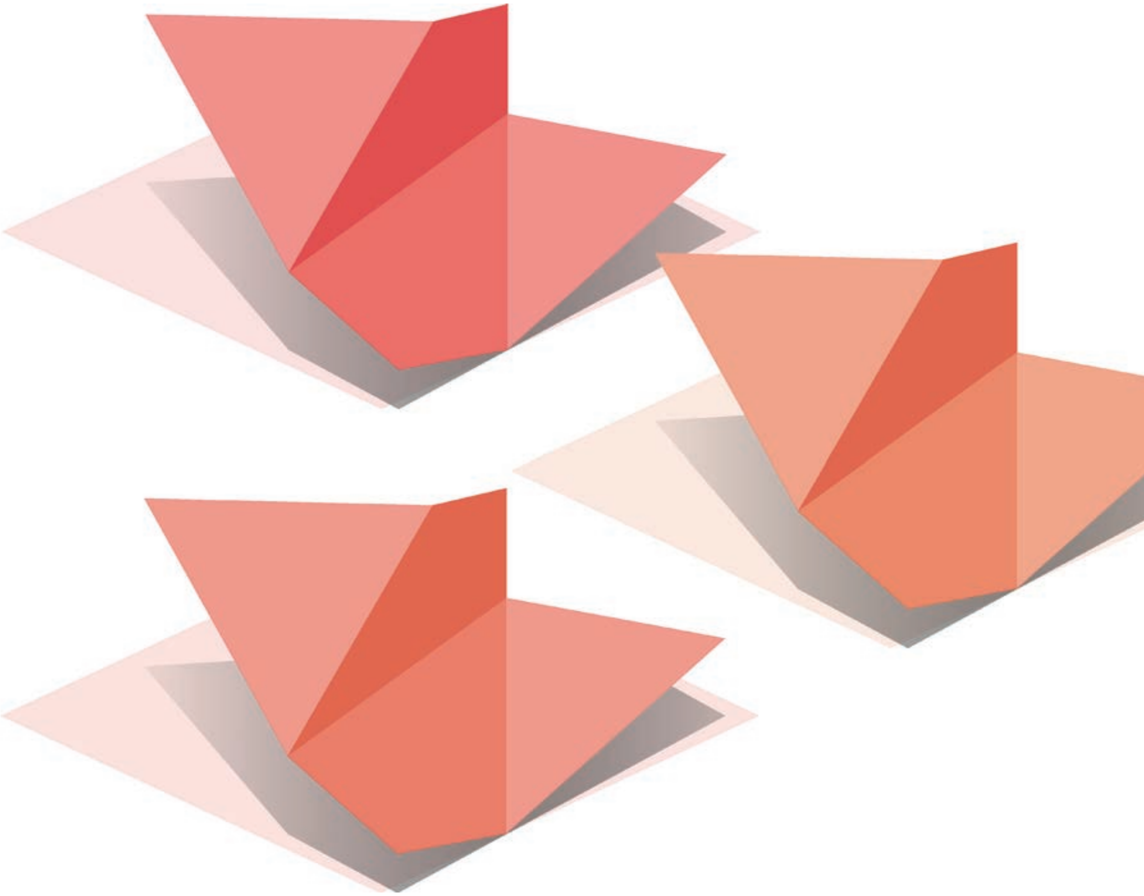

Andrea Casale, Graziano Mario Valenti,
Michele Calvano, Alfonso Oliva, Mia Tsiamis

The shape of the folded surfaces

drawing control and analysis

FORME DEL DISEGNO
FrancoAngeli

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The FORME DEL DISEGNO series is an opportunity to share reflections on design as an anthropologically natural language, both cultural and universal at the same time, which indicates simultaneously the conception and execution of its objects.
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The series is divided into three sections: PUNTO, which collects more purely theoretical contributions on specific issues, LINE, which hosts contributions aimed at the systematization of knowledge around specific topics, SURFACE, which presents experimental practices and activities on case of studies or peculiar topics.

Graphic project
Andrea Casale

Andrea Casale, Graziano Mario Valenti,
Michele Calvano, Alfonso Oliva, Mia Tsiamis

The shape of the folded surfaces
drawing control and analysis

FORME DEL DISEGNO
Sezione
LINEA

FrancoAngeli

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Andrea Casale, Graziano Mario Valenti and Michele Calvano shared the editorial project, taking care of the theoretical approach and the solutions of the geometry problems exposed in the first 9 chapters of the book.

The experimental application part was curated by Andrea Casale for the chapters on the articulated folded surface and by Graziano Mario Valenti for the chapters on the developable folded surfaces.

The experimental theoretical and applicative approach for the realization of the parametric procedures was curated by Michele Calvano for the chapters on the folded surface and by Graziano Mario Valenti for the chapters on the developable folded surfaces.

Alfonso Oliva and Mia Tsiamis have integrated the editorial project, curating the theoretical approach and experimental application solutions of chapters 10.

From a strictly editorial point of view: Andrea Casale wrote the chapters: 1,2,3,4,5 (pp.15-124); Michele Calvano chapters 6,7,8 (pp.125-174); Graziano Mario Valenti Chapter 9 (pp. 175-216); Alfonso Oliva and Mia Tsiamis chapter 10 (pp. 217-260).

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Foreword

Massimiliano Lo Turco

In the famous Parametricism as Style - Parametricist Manifesto, presented at the Venice Biennale in 2008, Patrick Schumacher supported parametric design in all its forms, arguing that it is, by now, "… penetrating into all corners of the discipline. Systematic, adaptive variation, continuous differentiation (rather than mere variety), and dynamic, parametric figuration concerns all design tasks, from urbanism to the level of tectonic detail, interior furnishings, and the world of products…" But the relationship between mathematics and descriptive geometry is not considered an innovative topic: the cultural origins of this movement can be found about forty years earlier. The wish to give rigor to form generation is the main reason of the initiative taken in the post-war period by Luigi Moretti who, together with Professor Bruno de Finetti, founded the Institute for Mathematical Operative Research in Architecture and Urban Design (IRMOU). That was the official starting point of the research around the subject of Parametric Architecture defined at the time. At the architectural scale, the group produced designs of stadium for soccer, tennis, swimming or movie theatres, that all followed a similar procedure. The research group starts from an analysis of the topic, that is then followed by the formulation of the needs, as geometrical relations between the quantities defined as parameters.

The book The shape of the folded surfaces *does not explicitly mention the terms parameter/parametric (except for the last parts), although it investigates and critically reflects on the possible spatial configurations that a surface can assume after creating the folds and creating a tessellation that facilitates its transformation. These relations are graphically explicit both through the use of geometric constructions and through the contribution of visual programming tools. This allows to establish dynamic relationships between data that describe a design in three dimensions, thereby making it possible to simultaneously address as a single entity what, until then, had been separated. Moreover, the title of the book does not mention the term origami, perhaps because of the risk of misunderstanding an in-depth research experience: the risk of confusing this work with an ancient art, often unfortunately associated with the creation not only of precise geometric patterns – as in the very interesting illustrated case studies – but with the creation of figures of people, objects, flowers, animals. This ancient technique is something more: a shining example that shows that scientific problems arise from everyday objects like a normal sheet of paper, in an original connection between art and geometry.*
Tomohiro Tachi, an Associate Professor in graphic and computer sciences at the Graduate School of Arts and Sciences, at the University of Tokyo is one of the leading experts in the study of the relationship between Origami and mathematics: At this regard, he states that "Origami allows us to build 3D functional shapes out of a 2D flat sheets. It further allows us to reprogram a single sheet pattern into different mechanisms and functions. However, the difficulty of the folding process rapidly grows as the complexity of the pattern increases". In other words, sequences of folds to prefigure the dynamism of the forms. Regarding to dynamism, the freedom (and the constraints!) of movement of the proposed models is also perceptible by a skilful organization of the iconographic apparatus, in which the time variable is expertly managed through the representation of different spatial configurations, based on the movement of the illustrated origami.
The iconographic apparatus therefore represents an added value because it makes the reasoning more explicit, organized according to levels of increasing complexity: the use of images is extremely heterogeneous and affects the different forms of representation that the authors propose: from the views in true form for the understanding of the axioms of Huzita-Hatori used in the introductory parts, to the three-dimensional models that report different configurations, to the physical models. Finally we can found the explicit definition of the algorithms able to virtually reconstruct not only the object resulting from

the bending operations, but also the logical process through which a form can be generated, function of constraints, tessellations and actions.
The common matrix is the Drawing, in its many forms, from whose reflection derives the title of the series in which the present volume is inserted in.
Another element of great interest is the sensibility to realize physical models in the described research works. There is no doubt that visual and tactile interactions with physical models offer a different channel for understanding structures and concepts. The increase in the number of possible tools for understanding and evaluating is useful in those areas of design that require non-linear processes.
Its applicability in the architectural field involves new professional figures, including computer scientists, artists, and engineers, through cross-disciplinary collaboration, as it is clear in the last chapters where the authors wonder about an interesting question: "Does fold follow function or does function follow fold?" The effect of fold pattern variations on the structural performance of folded geometry was evaluated using a specific workflow and computational methods were used to optimize the performance of structure. After the theoretical explorations, we could not miss the applications to the architectural field, less explicit in the previous chapters. The volume ends with interesting considerations deriving from the structural analyses that provide a framework for designing folded plate structures derived from Origami tessellations on an architectural scale.
The folded surfaces represent an extremely contemporary theme in architecture, in a period in which we are looking for discretization, through flat panels, of the complexity. The chapter on tessellation adds food for thought on the prefiguration of double-curved models using polyhedral structures with mostly identical faces: rethinking known patterns (Miura pattern) through the addition of a new class of folding, the neutral fold, allows the bent surface to describe double-curved forms.
The surfaces that make up the shells of contemporary architectures, develop subtly as clever shapes, chasing, folding and merging with elegance, with no apparent separation, and giving the illusion between before and after, between above and below. This richness, and simultaneously, this simplicity, is not only the outcome of technological evolution, but also has the underlying development, of more than a decade, of digital representation techniques based on mathematical modelling that have expanded the possibilities of managing complex forms for architects and designers. Complex forms have to be intended not only as curvilinear forms, but also as the integration of different sy-

stems (spatial, structural, coating, etc.) that constitutes the architectural object. In this operation, the creative act is of the utmost importance. Paraphrasing Henri Poincarè, a renowned mathematician, physicist, and French philosopher at the beginning of the 20th century, creativity consists of joining existing elements with new connections that are useful. In the most recent elaborations, reinforced by the new digital potentials, the main difference from the past is the shift of interest from the static balance to the search for dynamism through a change, recognisable, gradual, and continuous, of the basic scheme of repeatability of the pattern. And that is the motivation for the research activities proposed in the book.

Introduction

The study of the folded surface has ancient origins. With the term origami, which derives from the Japanese ori to fold and kami paper, we mean the art of folding the paper. This tradition of folding sheets of paper to produce figures can be traced back to the middle of the first millennium, when Buddhist monks imported paper in Japan. The rules of the game require that from a sheet of paper you get a shape with only the use of the fold, without glue or cuts. The skill lies in discovering all the possible forms deductible from a sheet of paper. Geometry is the first property observed in origami, but this is followed by symmetry, balance and proportion. Aesthetic, technical and geometric parameters that render the design of these surfaces particularly interesting for architecture, for engineering, and for design. However, two types of treatment of the folded surface must be distinguished. The first, linked to the tradition of this art, sees the use of the fold to get to simulate with the paper, shapes of animals, flowers and geometries also of considerable complexity. In these cases, the fold is created for reduce and shape the sheet of paper; this is sometimes overlapped and folded back on itself, other times it is stretched after the fold up until obtaining the desired shape. The second, instead, wants to investigate the properties of the fold and its ability to determine different spatial configurations on the surface. This is the kind of folding that this text wants to bring to attention, because it is particularly stimulating in the areas of architecture, design and even engineering. The problems faced in these experimental models could be solved more efficiently by using other paths provided by mathematics, but the geometric solution undoubtedly has the advantage of being more effective and generating creative ideas within the design process. In this perspective, the experimental use of the Visual Programming Language that operates in three-dimensional modeler, as well as facilitating the design of responsive architectures, favors the approach of the new generations of designers to the study of geometry. This is a significant aspect for the development of research and teaching in the field of drawing for the design.

1
The Huzita-Hatori axioms

In recent years, mathematicians and scientists have become increasingly interested in the properties of paper folding. Along the way, they have discovered new tools to go beyond the limits of traditional geometry. In fact, paper folding, has yielded solutions to several longstanding unsolved problems in geometry, such as trisecting an angle, doubling the cube, or constructing all of the regular polygons up to 21 sides.

A close connection with geometry can be made by folding a flat surface. Folds, are always straight, and folding one part of a sheet of paper onto itself is comparable to the result obtained by using a compass. Thus, any construction in Euclidian geometry that can be created with a straightedge and compass can also be found by folding a sheet of paper. Hence, Euclid's five postulates, are all relevant in paper folding. Drawing a line between two points corresponds to making a fold containing two points and, just like a line segment drawn with a straightedge, a fold can be extended indefinitely. Although it is impossible to make a circle of a given radius by folding a piece of paper, any point on a circle with a given radius can always be found. Orthogonal folds create equal right angles. Even Euclid's fifth postulate is applicable to paper folding: if a line intersects another two lines so that the sum of the internal angles on one side is less than two right angles, the two lines will eventually meet if extended far enough on that side. Finally, Playfair's axiom, which is commonly adopted by modern treatises, is also relevant: given a line and a point not on it, there is a unique line passing through the point that is parallel to the given line. All of these postulates are relevant to paper folding if we substitute the word *fold* for the word *line*.

At the First International Meeting of Origami Science and Technology – the first conference on the geometry of origami, held in Ferrara, Italy in 1989 – the mathematicians Humiaki Huzita and Benedetto Scimemi presented the first six axioms on which the mathematics of origami is based. The mathematician Koshiro Hatori added a seventh in 2002, and the axioms have since been known as the *Huzita-Hatori Axioms*.

As often occurs in research, the mathematician Jacques Justin had already published a paper in 1989 entitled "Résolution par le pliage de l'équation du troisième degré et applications géométriques," in which he stated seven possible relations capable of resolving the potential combinations of points and lines with single folds.

In the axioms, a point always results from the intersection of two folds and a line always corresponds to a fold. Two operations are therefore allowed: either the sheet of paper is simply folded over onto itself, or one part of the paper slides over the other until a solution is found to the given problem.

Indeed, it is the ability of the paper to slide over

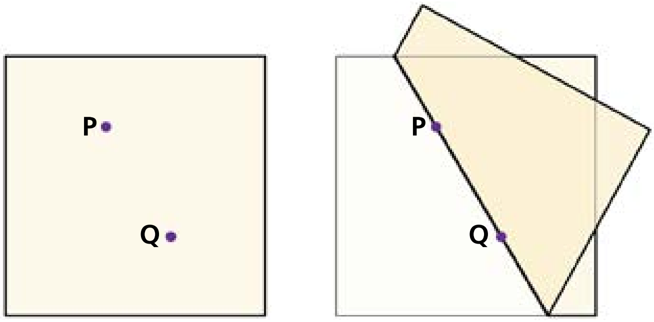


Fig. 1/ Given two points *P* and *Q*, there is a unique fold that passes through both of them.

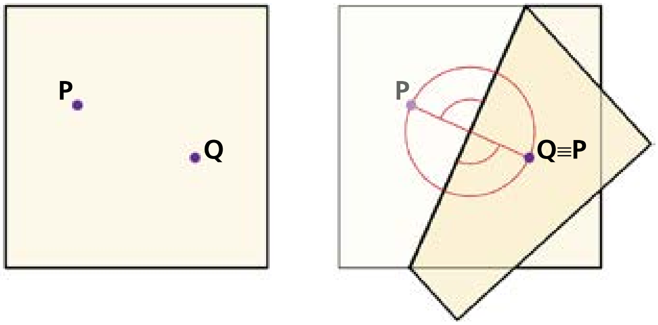
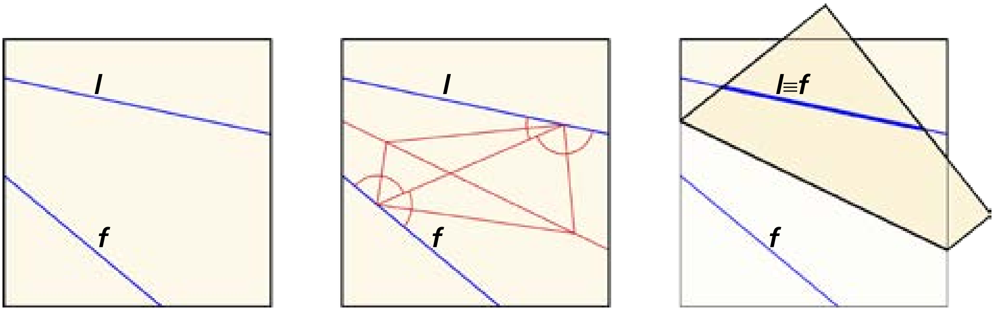


Fig. 2/ Given two points *P* and *Q*, there is a unique fold that places *P* onto *Q*.

itself that makes it so useful in solving problems that cannot be solved solely with a straightedge and compass. The following axioms synthetically present all the problems that may be encountered when constructing folded surfaces.

1 Given two points *P* and *Q*, there is a unique fold that passes through both of them (Fig. 1).
This operation is clear. Part of a sheet of paper is folded over so that the fold passes through the two given points, demonstrating Euclid's first postulate, which states, "*A unique line passes through any two points*".

2 Given two points *P* and *Q*, there is a unique fold that places *P* onto *Q* (Fig. 2).
Here, a part of a piece of paper is folded over so that one point lies on top of a second. This axiom is also easy to reconstruct: the fold is always the



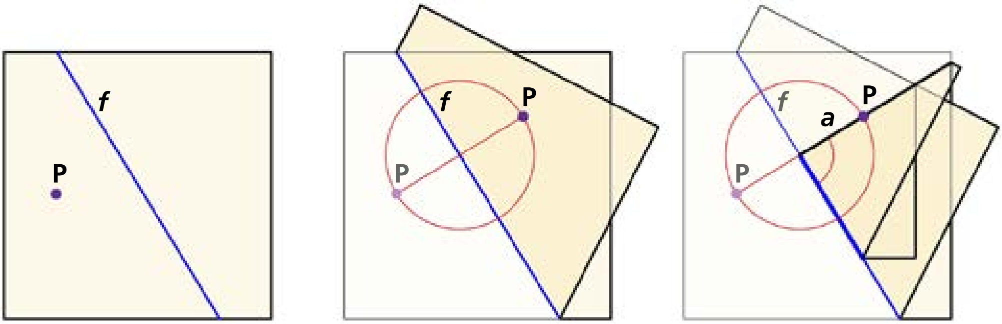


Fig. 3/ Given two straight lines *l* and *f*, there is a fold that places *l* onto *f*.

Fig. 4/ Given point *P* and line *f*, there is a unique fold perpendicular to f that passes through point *P*.

perpendicular line passing through the midpoint of the line connecting the two points.

3 Given two straight lines *l* and *f*, there is a fold that places *l* onto *f* (Fig. 3).
In this case, the two lines can be made to overlap by bringing them together and making the fold. The graphical representation of the problem shows that the fold bisects the angle between the two lines (Euclidian proposition 19).
This again suggests that since there are two bisectors between two (infinitely long) lines, there are two folds that solve the problem proposed by the axiom. There is only one solution if the two lines are parallel, in which case the fold is equidistant from the two.

4 Given point *P* and line *f*, there is a unique fold perpendicular to *f* that passes through point *P* (Fig. 4).

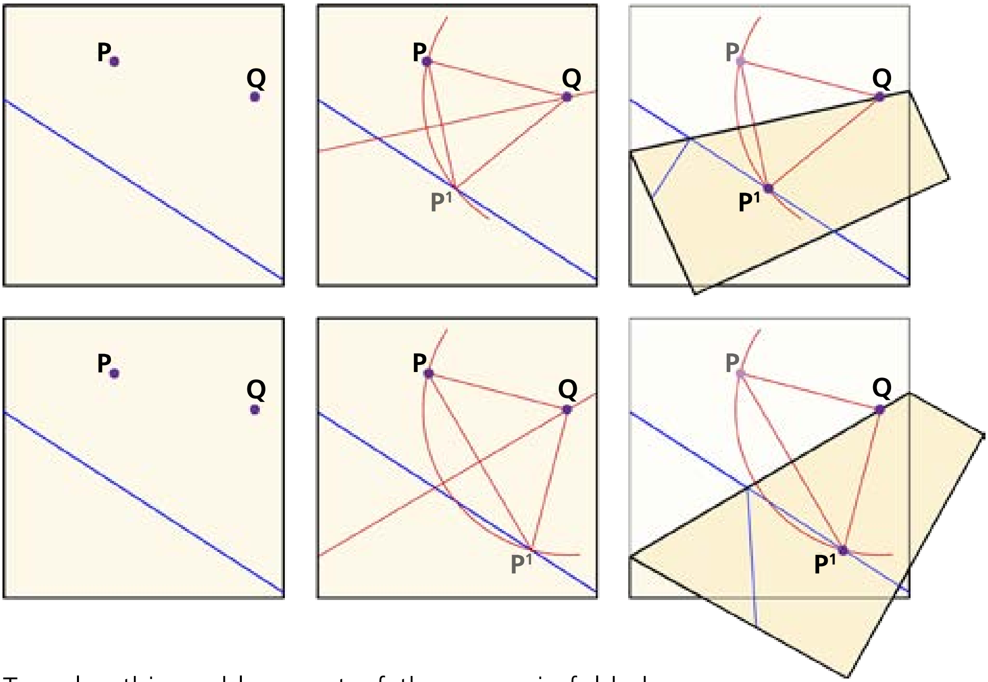


Fig. 5/ Given two points *P* and *Q* and a line *f*, there is a fold that places *P* onto *f* and passes through point *Q*.

To solve this problem part of the paper is folded over so that line ***f*** lies on top of itself, before being slid along ***f*** until the fold falls on point ***P***. In the geometrical construction, the perpendicular line ***a***, passing through point ***P***, solves the problem.

5 Given two points *P* and *Q* and a line *f*, there is a fold that places *P* onto *f* and passes through point *Q* (Fig. 5).

In practice, to find this solution, the paper is first folded over to make a crease that would pass through point *Q* and then rotated around point *Q* so that the part of the plane containing point *P* slides until it lies on line *f*.

Graphically, using a compass, a circle is made with its center at point *Q* and its radius equal to *QP*, and the point where the circle intersects line *f* is found, labeled *P1*. The bisector of the angle between lines *QP* and *QP1* is the fold *a* that places point *P* on line *f*.

Observing this, it is easy to see that there are three